BRAINWAVES REPORT BW/018[1]

THE LION, THE CAGE AND THE PEASHOOTER

> A man who could give a convincing account of mathematical reality would have solved very many of the most difficult problems of metaphysics. (G. H. Hardy, *A Mathematician's Apology*, p.123)

THE CONJURING TRICK

Religion today is getting a very bad press. To admit to being a Christian almost defines you as being if not downright eccentric, at least hopelessly out of touch with intellectual and scientific reality. The media are very much quicker to point out the Church's failures, its divisions, flaws and anticipated schisms, than they are its virtues and successes. London buses tell us not to worry because there probably isn't a God, as though He were some kind of enemy. In television dramas the representatives of faith appear most commonly as wimps and crackpots. The proponents of atheism and of a secular society are increasingly becoming celebrities.

I doubt if anyone encapsulates the mood of our age better than Richard Dawkins, in his bestselling book, *The God Delusion*, which caused a sensation when it came out in 2006. Yet to my way of thinking this is a profoundly unsatisfying book. This is not because as a Christian believer I cringe whenever Dawkins lands a punch on the misdeeds of the Church. Of course I do. Rather, what concerns me is that the God whom he attacks with such gusto is not actually the One in whom I believe. As such, his book comes across as a conjuring trick.

As every magician knows, the art of conjuring lies in directing one's audience away from the area where the real subtlety is taking place, pointing them instead somewhere else where what he is doing appears to be an impossibility. Dawkins is seemingly taking on a lion with his peashooter, which might be an act of real bravado were it not for the fact that his lion has already been caged before the book opens. The real subtlety lies in the question he does not raise: how does one get the lion into the cage in the first place? This is the real issue in the debate over the existence of God, and Dawkins does not just leave it unanswered; he studiously avoids asking it.

In fact, the question of the existence or non-existence of God belongs to the discipline of philosophy, where I judge it is the single most important issue. It is I suspect because Dawkins is on his own admission 'a scientist rather than a philosopher' (p.82) that he is somewhat out of his depth.

What Dawkins's God lacks is *mystery*. Mystery is a cluster of concepts including awesomeness and, lying beyond our comprehension. In religion it is defined by the *Shorter Oxford Dictionary* as 'a doctrine of faith involving difficulties which human reason is incapable of solving.' I believe that this includes the combining of apparent opposites. For such a thing Dawkins, with his unshakeable faith in science and the power of human reason, has no time. The God in whom he disbelieves is not allowed to lie beyond his comprehension. If He did, He would be immune to his

[1] This report develops some thoughts that were first introduced in the talk recorded as BW/004 'God, Maths and Plato'. For comments and criticisms I am indebted to my wife Barbara, to Fr Nicholas King, SJ, Richard H. C. Lee, Peter F. Van Peborgh, and Dr Watcyn Wynn.

onslaughts.

In what sense is the God of Christian theology a mystery? I offer one answer. Since Hume it has been a commonplace of philosophy to classify true propositions as either *necessary* or *contingent*. Broadly speaking they are called necessary if they are truths of reason, which are true in virtue of their meaning. You only have to think about them to know they are correct. They are not supposed to tell you anything about the 'real' world. For instance, 'A bachelor is an unmarried man.' If you understand the terms 'bachelor' and 'unmarried man' no further observation or perception is required to enable one to appreciate its truth. Contingent truths, on the other hand, state facts about the world which just 'happen' to be true. You verify them empirically, by observation, by investigation, by the methods of science and so forth. The distinction could be otherwise expressed as the difference between the conceptual and the material; or between what goes on inside the head and what goes on outside it. These two types of proposition clearly contrast. They lie at opposite poles. The question then arises, are they totally disjoint or can the apparent disjunction be bridged?

Dawkins's God, '*a superhuman intelligence who deliberately designed and created the universe and everything in it, including us*' (p.31), is unashamedly contingent. As Dawkins insists throughout, He is a scientific hypothesis and therefore amenable to scientific investigation which alone is competent to determine His existence or otherwise.

But what if a stronger claim were made, that God is both contingent and necessary? This I believe to be the implication of Christian theology, in which God embodies the truth in all cognitive domains. Then the real issue to be determined is a philosophical and indeed metaphysical one. Can anything or anyone be both necessary and contingent? It seems to be a contradiction in terms, *unless we allow the category of mystery to be valid*. Is mystery a non-empty set? Is there precedent anywhere else for apparently contrasting opposites both to be true?

In fact there is a ready candidate within physics itself. In the seventeenth century there were two rival candidates for the nature of light. Newton held that light was a stream of 'corpuscles' or particles. Huygens maintained that it was a wave motion. The two views were seemingly irreconcilable. Today we learn that light has a dual nature, both wave and particle. This may appear self-contradictory and certainly lies beyond visualisation, but is now a well-established fact. We should not dismiss too quickly seemingly irreconcilable opposites.

However the possibility of mystery is something Dawkins does not address. His God is non-mysterious because the possibility that God might be a necessary truth does not occur to him. So when we open his book we find that it is against a caged lion that he directs his peashooter. Once we buy into that the trick is complete.

That Dawkins's problem lies in his misdefinition of God is well grasped by the Marxist writer Terry Eagleton in his book *Reason, Faith and Revolution*. Describing 'the so-called new theology' which he first encountered at the age of eighteen, he writes:

> It did not see God the Creator as some kind of mega-manufacturer or cosmic chief officer, as the Richard Dawkins school of nineteenth-century liberal rationalism tends to imagine - what the theologian Herbert McCabe calls "the idolatrous notion of God as a very powerful creature." Dawkins falsely considers that Christianity offers a rival view of the universe to science. Like the philosopher Daniel C. Dennett in *Breaking*

> *the Spell*, he thinks it is a kind of bogus theory or pseudo-explanation of the world. In this sense, he is rather like someone who thinks that a novel is a botched piece of sociology, and who therefore can't see the point of it at all. Why bother with Robert Musil when you can read Max Weber? (p.6)

> God for Christian theology is not a mega-manufacturer. He is rather what sustains all things in being by his love, and would still be this even if the world had no beginning. Creation is not about getting things off the ground. Rather, God is the reason why there is something rather than nothing, the condition of possibility of any entity whatsoever. Not being any sort of entity himself, however, he is not to be reckoned up alongside these things, any more than my envy and my left foot constitute a pair of objects. God and the universe do not make two. (pp.7-8)

So the question before us is, Is there or is there not mystery, in the sense in which I have defined it: the possibility that anything can have seemingly opposite characteristics in a way that is beyond our comprehension without losing its identity? Specifically, can anything be both necessary and contingent at the same time? If there is no mystery, then there is no Christian God; there is no more to be said, and Dawkins's book is redundant. If there is mystery, he is wrong.

THE MEANINGLESSNESS OF A. J. AYER

Is there mystery? The question which Dawkins ducked has been tackled in one guise or another by numerous philosophers down the ages, but seldom with more aplomb than it was in 1936 by an angry young man called A. J. Ayer, later Sir Alfred Ayer, Wykeham Professor of Logic at the University of Oxford, in his acclaimed book *Language, Truth and Logic*. This formed a sort of manifesto for the logical positivist school that originated with the Vienna Circle in the 1920s.

Ayer's plan was one of breathtaking boldness. As he explained in his Introduction,

> Like Hume, I divide all genuine propositions into two classes: those which, in his terminology, concern "relations of ideas," and those which concern "matters of fact." The former class comprises the *a priori* propositions of logic and pure mathematics, and these I allow to be necessary and certain only because they are analytic. That is, I maintain that the reason why these propositions cannot be confuted in experience is that they do not make any assertion about the empirical world, but simply record our determination to use symbols in a certain fashion. Propositions concerning empirical matters of fact, on the other hand, I hold to be hypotheses, which can be probable but never certain.[2] And in giving an account of the method of their validation I claim also to have explained the nature of truth. (p.31 of the 1946 edition.)

That would be some achievement for a twenty-six year old. He continued:

> To test whether a sentence expresses a genuine hypothesis, I adopt what may be called a modified verification principle. For I require of an empirical hypothesis, not indeed that it should be conclusively verifiable, but that some possible sense-experience should be relevant to the determination of its truth or falsehood. If a putative proposition fails to satisfy this principle, and is not a tautology, then I hold that it is metaphysical, and that, being metaphysical, it is neither true nor false but literally senseless. It will be found that much of what ordinarily passes for philosophy is metaphysical according to this criterion, and, in particular, that it could not be significantly asserted that there is a non-empirical world of values, or that men have immortal souls, or that there is a transcendent God. (p.31)

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[2] Ayer has wrongly understood the word 'hypothesis'. A proposition may be entertained in several different ways: for instance affirmatively, as 'I am a mountain goat'; interrogatively, as 'Am I a mountain goat?'; conditionally, as 'If I were a mountain goat'; or optatively, as 'Would I were a mountain goat'. Of these, only the third is a hypothesis. The affirmation, 'I am a mountain goat' is not a hypothesis but a statement which in most cases is unlikely to be true. Linguistic analysis had not got very far when Ayer was writing.

Ayer's 'metaphysics' has a considerable overlap with what we have called mystery and we may fairly consider that the two stand or fall together. If he wins his case, then the lion is caged. But Ayer himself misunderstood the strengths and weaknesses of his own position. Anything which fails his test of verification, such as in his view metaphysics and religion, he stigmatizes as not false but *meaningless*. But the verification principle, even in the modified sense in which Ayer employs it (i.e. by not requiring conclusive verification), soon runs into logical difficulties. How can it be verified? In default of any means of verifying it - and Ayer proposes none - the verification principle itself fails. However what in fact would abolish God is not the verification principle but the dichotomy he makes between "matters of fact" (the contingent) and the truths of logic and pure mathematics (the necessary or tautological). It is this dichotomy which, if made good, would cage the lion. But can it be done?

Ayer's empiricist stance about the necessary and the contingent entails two propositions:

> (1) That thought alone can tell us nothing about the outside 'real' world. This amounts to a denial of intuition.
> (2) That logic and mathematics are either not necessary, or else have no factual content. (p.73. He opts for the latter.)

I shall argue that the first of these is refuted by history; the second, by mathematics.

THE FACT OF INTUITION

The unifying thread that runs through Ayer's book is the denial of the possibility of intuition (whose philosophical sense is defined by the *Shorter Oxford Dictionary* as the 'immediate apprehension by the intellect alone'). He denies emphatically the rationalist view that 'there are some truths about the world that we can know independently of experience' (p.73), which concession to rationalism, if granted,

> would upset the main argument of this book....And thus the whole force of our attack on metaphysics would be destroyed....For the fundamental tenet of rationalism is that thought is an independent source of knowledge. (Ibid.)

However most of us who are not dogmatic empiricists have no difficulty with intuition, whether or not we commonly experience it ourselves.

Ayer is thus setting himself up for a fall. Consider for instance the report by Simon De Bruxelles in *The Times* of 11 December 1998 headed 'Dream guided treasure hunter to Roman coins,' of Mr Colin Roberts who after searching a field twice with his metal detector without success, on subsequently dreaming twice of a big find there, returned and found a hoard of 3,778 Roman coins. As he told the inquest,

> In my dream, I could see myself in the middle of the field pulling up a haul of coins. When I had the same dream a few nights later, I took a few hours off work next day. Normally, I would start by the gate and work across but this time I went to the middle. I took two paces and my detector bleeped.

There is also the phenomenon of prophecy. As I have argued elsewhere,[3] this seems to be a gift exercised by a small number of great men at different points in history. As examples I suggested Jeremiah, Jesus of Nazareth, C. G. Jung and Sir Winston Churchill. In the latter case I quoted from Professor Niall Ferguson's book *Empire*:

> I can see vast changes coming over a now peaceful world; great upheavals, terrible struggles; wars such as one cannot imagine; and I tell you London will be in danger - London will be attacked and I shall be very prominent in the defence of London...I see further ahead than you do. I see into the future. The country will be subjected somehow to a tremendous invasion...but I tell you I shall be in command of the defences of London and I shall save London and the Empire from disaster.[4]

According to Ferguson, 'Winston Churchill was just sixteen when he spoke those words to a fellow Harrovian, Murland Evans.'

The significant point is that, whether these or any other individual instances be granted, the issue of whether or not one can learn matters of fact from within one's own head - be it by intuition, dreams, extrasensory perception or anyhow - is an empirical one, to be determined on a case by case basis and cannot be predetermined by logical argument as attempted by Ayer. As we know from Jung, the unconscious is a powerful thing, and we ignore it at our cost.

Ayer in fact seems quite unaware of the role played by intuition repeatedly in the advancement of science. Antony Bridge, who was Dean of Guildford Cathedral from 1968, put it in his autobiography, *One Man's Advent*, as follows:

> One of the most celebrated and dramatic examples of such a moment of illumination - such a moment of disclosure - was that experienced by Otto Loewi, the Professor of Pharmacology at the University of Graz. He had been puzzling for a long time over the mechanism by which nerves affect muscles; one night he woke up with a brilliant idea, and reaching for a piece of paper he jotted down a few notes. In the morning he found to his dismay that they were illegible, while he had completely forgotten the idea which had come to him so vividly a few hours previously. All day he struggled to recall it, but it refused to come back. He went to bed depressed; then in the middle of the night he woke again with the same flash of inspiration - the same clue to the problem - and this time he wrote it down with great care. In the morning, when he checked it, he found that it was indeed the vital clue to the understanding of the process of chemical intermediation, not only between nerves and muscles and the glands they affect, but also between the nervous elements themselves; and startling as such a story might be, the biographies of scientists are full of similar tales. 'I can remember the very spot on the road, whilst in my carriage, when to my joy the solution occurred to me,' wrote Charles Darwin of the moment when the idea that natural selection was responsible for the origin and development of species came to him in a flash and changed the course of science and human thinking. Like Otto Loewi, it was in his sleep that Otto Kekulé dreamed of the benzene ring, while Semelweiss's discovery of the cause of childbed fever, Kepler's idea about the elliptical orbit of the planets around the sun, Pasteur's discovery of the cause of anthrax, Einstein's famous intuition which led him to formulate the theory of relativity and hundreds of other discoveries and clues to the solutions of intractable scientific problems are on record as 'coming' to the investigators in moments of illumination. It is no wonder that Professor Hawkinge [*sic*] of Cambridge said recently on television that the one thing necessary to become a good physicist was to have the ability to make intutive leaps. Roger Penrose, Professor of Mathematics at Oxford and the inventor of a new, post-Einsteinian, six-dimensional space, which he calls 'twister space', has emphasised even more strongly the central part played in creative science by intuition and the authoritative place of aesthetic satisfaction in the acceptability of mathematical theory. (pp.61-62)

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[3] Martin Mosse, *The Three Gospels*, pp.186-95.

[4] Churchill. Quoted in Ferguson, *Empire*, p.292.

The reference to Einstein requires some elaboration. As Michael Polanyi explains in his singular work, *Personal Knowledge*, in the textbook account, Einstein posited the special theory of relativity to explain the failure of the Michelson-Morley experiment of 1887 to identify 'ether' drift - that is, they allegedly found that the speed of light measured by a terrestrial observer was the same in whatever direction the signal was sent out. This led Einstein to abandon the Newtonian concept of time and space, with its sense of absolute motion and absolute rest, in favour of a new conception of space-time in which only the relative motion of bodies could be expressed. 'But the historical facts,' says Polanyi (p.10), 'are different.'

> Einstein had speculated already as a schoolboy, at the age of sixteen, on the curious consequences that would occur if an observer pursued and kept pace with a light signal sent out by him. His autobiography reveals that he discovered relativity
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>> after ten years' reflection...from a paradox upon which I had already hit at the age of sixteen: If I pursue a beam of light at the velocity of *c* (velocity of light in a vacuum), I should observe such a beam of light as a spatially oscillatory electromagnetic field at rest. However, there seems to be no such thing, whether on the basis of experience or according to Maxwell's equations. From the beginning it appeared to me intuitively clear that, judged from the standpoint of such an observer, everything would have to happen according to the same laws as for an observer who, relative to the earth, was at rest.[5]
>
> There is no mention here of the Michelson-Morley experiment. Its findings were, on the basis of pure speculation, rationally intuited by Einstein before he had ever heard about it. To make sure of this, I addressed an enquiry to the late Professor Einstein, who confirmed the fact that 'the Michelson-Morley experiment had a negligible effect on the discovery of relativity.' (*Personal Knowledge* p.10)

Polanyi goes on to note that the Michelson-Morley experiment did not in fact give the result required by relativity!

> It admittedly substantiated its authors' claim that the relative motion of the earth and the 'ether' did not exceed a quarter of the earth's velocity. But the actually observed effect was not negligible; or has, at any rate, not been proved negligible up to this day [1962]. (Ibid. p.12)

The truth, it seems, is a little stranger than Ayer conceived it to be. His emphatic claim that 'there can be no *a priori* knowledge of reality' (p.86) fails upon examination. The empiricism to which he adheres does not work in practice. Thought *can* be an independent source of knowledge. But he has explicitly tied to it the success or failure of his attack upon the entire body of metaphysics, including belief in God. So under the rules of engagement which he himself defined, his attack upon God fails. The lion therefore is poised to leave his cage.

Ayer falls down because he chooses to ground his philosophy upon the nature of *propositions*. As a result he is wholly ill-equipped to give an adequate account of the extraordinary phenomenon of *people* as thinking beings. *A fortiori*, he is inevitably going to miss out on any God who is personal. His starting categories do not permit him to do otherwise. Contrast Polanyi, whose raw material is people, and whose epistemology in *Personal Knowledge* turns out to be very much more robust than Ayer's.

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[5] *Albert Einstein: Philosopher-Scientist*, Evanston, 1949, p.53.

THE FACTUAL CONTENT OF MATHEMATICS

Having now demonstrated the fact of intuition, we turn to Ayer's second contention, that mathematics has no factual content. Our aim will be to show that mathematics seamlessly bridges the divide between the necessary and the contingent, thus qualifying for the title 'mystery'. We shall argue that

(a) Mathematics cannot be reduced to tautology;
(b) Mathematics provides the basis of science; and
(c) The constants of mathematics are fundamental to the universe.

In so doing we shall propose that there are significant parallels between mathematics and the God of the traditional monotheisms. So, many of the standard objections to belief - for instance that He cannot be observed by the instruments and methods of science - prove to be invalid because they would equally well condemn the entire body of mathematics.

(a) Mathematics and Tautology

It is Ayer's belief that 'the truths of logic and mathematics are analytic propositions or tautologies.' (p.77) The word analytic requires some explanation. It harks back to Kant's *Critique of Pure Reason* (1781) in which he makes a pair of distinctions, similar to that we have already encountered between the necessary and the contingent. The first, between the analytic and the synthetic, concerns the *logical status* of propositions. They are

analytic, if the predicate concept is contained in the subject concept, as
'All bachelors are unmarried';
synthetic, if it is not so contained, as
'All bachelors are unhappy.'

Kant's second distinction between propositions refers to *the way we come to know them*. They are

a priori, if their justification does not rely on experience, as
'7 + 5 = 12';
a posteriori (or empirical), if their justification does rely on experience, as
'Some tables are red.'

Kant considered all possible permutations of these two distinctions, rejecting the possibility of the analytic *a posteriori*, but seeing in the synthetic *a priori* a possible home for metaphysics and mathematics. This has not won him many friends and it is hard to resist the conclusion, with Ayer, that he is overcomplicating things. Not much is lost if we regard as coterminous the necessary, the analytic and the *a priori* on the one hand; and the contingent, the synthetic and the *a posteriori* on the other. Nevertheless, the possibility that mathematics has a foot in both camps will prove worth pondering.

In support of his belief that 'the truths of logic and mathematics are analytic propositions or tautologies,' Ayer quotes Poincaré with approval (p.85):

If all the assertions which mathematics puts forward can be derived from one another by formal logic,

> mathematics cannot amount to anything more than an immense tautology.[6]

Even if we confine ourselves to pure mathematics this is incorrect. It is true that some equations are identities, which gain their force from being tautological. An example is

$$x^2 - y^2 = (x + y)(x - y)$$

in which the right hand side is simply a restatement of the left hand side, and which is true for all values of *x* and *y*. But not all equations are identities. The mistake is similar to describing all contingent statements as hypotheses (see n.2).

As to the proposal to derive the whole of mathematics from a set of axioms by formal logic, this was Hilbert's (1862-1943) programme, put forward by him at the start of the twentieth century. It led to Russell and Whitehead's *Principia Mathematica* (1910-13), but was dealt a mortal blow by Gödel's incompleteness theorems (1931), according to which in any axiomatic system containing arithmetic there will always be some propositions which cannot be proven either true or false.[7] This being so, the reduction of mathematics to a tautological restatement of axioms cannot be made good.

(b) Mathematics the Basis of Science

Mathematics is infinitely richer than Ayer supposed. The most obvious objection to his thesis is that he gives no account whatever of applied mathematics, which seamlessly spans the gap between the necessary and the contingent. This includes for instance the whole of statistics, such as notably the ubiquitous 'bell' curve of normal (Gaussian) probability, which supplies the basis of scientific research in countless fields.

Ayer gives no adequate account of why, if mathematics has no meaning or reference to the real world, *applied mathematics almost universally forms the basis of science*. And time and again, applied mathematics starts off life as pure. Thus i, the square root of minus one, was first posited by the Italian mathematicians of the sixteenth century as a means of solving cubic equations. Today it is indispensable to the theory of alternating electrical currents. Since then, when scientists want to begin a conceptual revolution, they frequently look for cutting-edge mathematics in order to describe it. So Einstein based his general theory of relativity on non-Euclidean geometry which had been developed in the previous century, and in particular by Bernhard Riemann in 1854. When Heisenberg in 1925 needed a non-commutative multiplication to support quantum mechanics, he turned to matrices (developed by the Englishman Arthur Cayley in 1858). Group theory, founded by Galois in 1829, today provides a framework for the development of particle physics. Fourier series, developed by Joseph Fourier in 1822, are foundational to waveform analysis. Fractals, investigated by Mandelbrot in 1980, which exhibit self-similarity by which a pattern recurs independently of magnification, appear frequently in nature - for instance in the shape of trees, or coastlines, or in the formation of crystals. The

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[6] J. H. Poincaré (1854-1912), *La Science et l'Hypothèse*, Part I, Chapter i.

[7] An example is the continuum hypothesis about the relative magnitudes of different types of infinity. It has been shown that on the basis of the axioms of formal set theory this can be neither proved (Gödel, 1937) nor disproved (Cohen, 1964). The Goldbach conjecture - that every even number greater than 2 is the sum of two primes - has been suggested as another candidate. Most mathematicians regard it as highly probable - it has been tested by computer to some enormously high numbers - but since it was first put forward in 1742, no one has been able to prove it or disprove it.

Poincaré conjecture in topology,[8] first mooted by Poincaré in 1904 and finally proved by Perelman in 2002-3, has much to teach us about the shape of the universe. It appears also that there is a fruitful correspondence between the energy levels in certain heavy nuclei and the location of the zeta function zeros being investigated as tests of the Riemann hypothesis - that is, between the respective cutting edges of nuclear physics and pure mathematics.[9]

Again, G. H. Hardy in another famous passage reflected in 1940 on his life's work as a pure mathematician:

> I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world. (*A Mathematician's Apology*, p.150)

But his field of number theory has subsequently provided the basis for modern cryptography which in turn supports the internet. We use it every time we check our online bank statement or purchase a book from an online retailer.

(c) The Constants of Mathematics

One of the surprising facts about mathematics that the student encounters is the remarkable prominence of a small number of special constants. The first one that we meet - and the first to be encountered by the human race - is π, which is introduced to us as the ratio of the circumference of a circle - any circle - to its diameter. Its approximate value is 3.14159. But π occurs in an astonishing number of different contexts throughout mathematics, for instance as the subject of innumerable expressions and series, some simple, some bizarrely complicated. Next, and of comparable importance, is *e*, the base of natural logarithms (approximately 2.718) which is also essential to the Calculus. There are others which don't appear quite as widely, such as ϕ, the golden ratio, about 1.618, closely associated with the Fibonacci sequence (1, 1, 2, 3, 5, 8,...) and γ (Euler's constant, about 0.5772) which periodically crops up in number theory. Of a different kind but still of phenomenal importance is i, defined above. Without all these, the amount of mathematics we could do would be severely circumscribed. There have been whole books written about each of them.[10]

Euler in 1748 demonstrated the unity of the 'core mathematics' of his day: trigonometry (founded upon π), complex numbers (founded upon i), and exponentials and logarithms (both founded upon *e*), together with the infinite series by which these various functions are calculated. He brought them together with his celebrated identities

$$e^{ix} = \cos x + i \sin x;$$

[8] Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.

[9] See M. du Sautoy, *The Music of the Primes*, Chapter 11, 'From Orderly Zeros to Quantum Chaos,' and Dan Rockmore, *Stalking the Reimann Hypothesis*, Chapter 11, 'A Chance Meeting of Two Minds'.

[10] π: Petr Beckmann, *A History of π*; see also David Blatner's delightful little compendium, *The Joy of π.*

e: Eli Maor, *e: The Story of a Number*. There is also an entertaining introduction in Martin Gardner, *Further Mathematical Diversions*, Chapter 3, 'The Transcendental Number *e*', pp.34-42.

ϕ: Mario Livio, *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number;* H. E. Huntley, *The Divine Proportion: A Study in Mathematical Beauty*.

γ: Julian Havil, *Gamma: Euler's Constant*.

i: Paul J. Nahin, *An Imaginary Tale: The Story of $\sqrt{-1}$.*

$$e^{-ix} = \cos x - i \sin x.$$

This was one of the greatest unifying achievements in mathematics of all time, comparable to Newton's unifying of Kepler's three laws describing planetary motion by his single law of gravitation.[11]

Now the remarkable thing is this. Every particle and every wave pulse everywhere and at all times in the universe exemplify the core truths represented in Euler's identities, just as every cell in our bodies exhibits our personal DNA. We have just seen that the constants *e* and π are essential to mathematics. It is now clear that they are essential to physics as well. Without them no physical universe would be even imaginable. So Rouse Ball and Coxeter write:

> I recall a distinguished professor explaining how different would be the ordinary life of a race of beings for whom the fundamental processes of arithmetic, algebra and geometry were different from those which seem to us to be so evident; but, he added, it is impossible to conceive of a universe in which *e* and π should not exist.[12]

(Contrast the physical constants such as Planck's constant, *h*, found in the study of the atom, its nucleus and its radiation. We could at least imagine a universe in which gravity or the other physical forces had different strengths, even if it did not last very long or could not produce life.) To this Kasner and Newman riposted (*Mathematics and the Imagination*, p.89):

> A universe in which *e* and π were lacking, would not, as some anthropomorphic soul has said, be inconceivable. One could hardly imagine that the sun would fail to rise, or the tides cease to flow for lack of *e* and π. But without these mathematical artifacts, what we know about the sun and the tides, indeed our ability to describe all natural phenomena, physical, biological, chemical or statistical, would be reduced to primitive dimensions.

But the sunrise and the tides are periodic events, and as such conform to Euler's identities, which lie at the heart of all periodic functions. And we have no control whatever over *e* and π. To suggest that they are 'artifacts' - human creations - of any kind is to be guilty of the same anthropomorphisation of which they are complaining. So *mathematics, so far from being a collection of meaningless tautologies, is essential to the very existence of the universe*. This completes our case for the factual content of mathematics.

MATHEMATICS AS MYSTERY

There is therefore no hard and fast distinction between pure and applied mathematics. Today's pure mathematics becomes tomorrow's raw science. This is a contingent fact about the nature of the universe as much as it is about mathematics itself. There is a seamless connection between pure mathematics and the physical sciences - between the necessary and the contingent - which Ayer cannot explain. His attempt to divorce the two, which formed the basis of his attack on metaphysics in general and God in particular, cannot therefore be made good. *There is mystery*. That being so, Dawkins's fundamental assumption is vitiated.

This mystery of mathematics is well pinpointed by Gian-Carlo Rota:

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[11] On the mathematical consequences of Euler's identities see Paul J. Nahin, *Dr. Euler's Fabulous Formula Cures Many Mathematical Ills.*

[12] W.W. Rouse Ball & H.S.M. Coxeter, *Mathematical Recreations and Essays*, p.348.

> 'The mystery of mathematics...is that conclusions originating in the play of the mind do have striking practical applications.'[13]

Or, as Einstein put it,

> 'How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?'[14]

Or the Physics Nobel laureate Eugene Wigner (1902-95):

> The miracle of the appropriateness of the language of mathematics to the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or worse, to our pleasure, even though perhaps to our bafflement, to wide branches of learning.[15]

Mathematics is on the one hand abstract, a set of concepts we apprehend and manipulate with our minds. On the other hand, its truths are external and independent of us, being inherent in the entire structure of the universe; the starting point of science. These two characteristics seem irreconcilable. So at the heart of mathematics lies a mystery: putting it another way, the category of mystery is a non-empty set. Dawkins's unwritten assumption therefore proves false. His lion is uncaged. There could be a God after all.

THE ONTOLOGY OF GOD

The foregoing argument in favour of mystery presents us with the possibility that God may exist. But in that case, how are we to understand 'exist'? We recall the ancient conundrum, how do we know that things exist when no one is perceiving them? - and Bishop Berkeley's answer that all things exist in the mind of God.[16] This gave rise to a celebrated limerick by Ronald Knox (1924), of which one variant reads,

> There once was a man who said, 'God,
> I find it exceedingly odd
> That the sycamore tree
> Should continue to be
> When there's no one about in the Quad.'[17]

To which came the equally celebrated anonymous reply,

> Dear Sir,
> Your astonishment's odd:
> *I* am always about in the Quad.

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[13] Gian-Carlo Rota, Introduction to Davis and Hersh, *The Mathematical Experience*, p.xix.

[14] Albert Einstein (1879-1955), quoted in J. Havil, *Gamma: Exploring Euler's Constant* , p.139.

[15] Quoted in M. Livio, *Is God a Mathematician?*, p.3.

[16] Bishop George Berkeley (1685-1753) maintained the idealist philosophical vew that *esse est percipi vel percipere* ('to be is to be perceived or a perceiver'). He also produced some strong and very just criticisms of the Calculus in the form in which Newton had cast it, which were not resolved until the development of analysis some 160 years later.

[17] Another variant of this limerick, with the reply, is attributed in a similar context to the great Hungarian number theorist Paul Erdös in his biography by Paul Hoffman, *The Man Who Loved Only Numbers*, p.26.

> And that's why the tree
> Will continue to be,
> Since observed by,
> Yours faithfully,
> God.

While this may be a little unconvincing in respect of material objects like tables and chairs and trees, it does give one pause for thought in respect of the objects of mathematics, and a clue as to their nature.

Here again, in the use of the word 'exist', mathematics provides us with a model. There is a long-running dispute as to whether mathematics is discovered or invented. Does it describe an abstract world which is 'really there', or does it simply consist in proving conclusions from sets of axioms? These two principal answers are respectively called Platonism, after its originator Plato (*c*.429-347 B.C.), and formalism.[18]

(a) Platonism

The view that numbers are abstract entities seems to have arisen with the school of Pythagoras. The view that all of mathematics is about absolutes that really exist was as far as we know first expressed by Plato, e.g.:

> And do you not know also that although they make use of the [geometrical] forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter, and so on - the forms which they draw and make, and which themselves have shadows and reflections in water, are in turn converted by them into images; for they are really seeking to behold the things themselves, which can only be seen with the eye of the mind?[19]

When not on the defensive against philosophers, most mathematicians speak, write and act as though they live corporately in a shared world of absolute truths which are there for them to discover and which are common ground between them. One celebrated expression of this is to be found in G. H. Hardy's testament, *A Mathematician's Apology*:

> For me, and I suppose for most mathematicians, there is another reality, which I will call "mathematical reality"....I believe that mathematical reality lies outside us, that our function is to discover or *observe* it, and that the theorems which we prove, and which we describe grandiloquently as our "creations", are simply the notes of our observations. This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards. (1992 edition, pp.123-4, emphasis original.)

Another comes from Kurt Gödel (1906-78):

> Despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception....They, too, may represent an aspect of objective reality.[20]

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[18] For two excellent discussions of this the reader is referred to Davis and Hersh, op. cit., and to M. Livio, *Is God a Mathematician?*

[19] Plato, *Republic* VI, 510d-e (tr. Jowett; cf. 527b).

[20] Quoted in Davis and Hersh, op. cit., p.319.

Or again, Sir Roger Penrose, Emeritus Rouse Ball Professor of Mathematics at the University of Oxford and Gresham Professor of Geometry at Gresham College, London:

> I cannot help feeling that, with mathematics, the case for believing in some kind of etherial, eternal existence, at least for the more profound mathematical concepts, is a good deal stronger than in those other cases [e.g. the arts]. There is a compelling uniqueness and universality in such mathematical ideas which seems to be of quite a different order from that which one could expect in the arts or engineering.[21]

Again:

> In my own mind, the absoluteness of mathematical truth and the Platonic existence of mathematical concepts are essentially the same thing. The 'existence' that must be attributed to the Mandelbrot set, for example, is a feature of its 'absolute' nature. Whether a point of the Argand plane does, or does not, belong to the Mandelbrot set is an absolute question, independent of which mathematician, or which computer, is examining it. It is the Mandelbrot set's 'mathematician-independence' that gives it its Platonic existence.[22]

Or Professor Alain Connes, winner of two of the most prestigious prizes in mathematics, the Fields Medal (1982) and the Crafoord Prize (2001):

> Take prime numbers [those divisible only by one and themselves], for example, which, as far as I''m concerned, constitute a more stable reality than the material reality that surrounds us. The working mathematician can be likened to an explorer who sets out to discover the world. One discovers basic facts from experience. In doing simple calculations, for example, one realizes that the series of prime numbers seems to go on without end. The mathematician's job, then, is to demonstrate that there exists an infinity of prime numbers. This is, of course, an old result due to Euclid. One of the most interesting consequences of this proof is that if someone claims one day to have found the greatest prime number, it will be easy to show that he's wrong. The same is true for any proof. We run up therefore against a reality every bit as incontestable as physical reality.[23]

So in practice, *most mathematicians are Platonists most of the time*. They have no difficulty in believing in, and reasoning about, an abstract world which is every bit as real as - Plato would say, even more real than - the everyday world of sense perception.

(b) Formalism

When challenged to justify their Platonism by modern day philosophers, mathematicians often get embarrassed, taking refuge in *formalism*: This is the doctrine that mathematics is *no more than* the process of deducing conclusions from premises - proving *theorems* from *axioms* - which lies behind Ayer's view that mathematics consists of tautologies. The axioms themselves are generally held to be either self-evident or otherwise chosen for their mathematical convenience. So starting from the geometrical axioms of Euclid (*c*.300 BC) we can prove Pythagoras' theorem. If we started with different axioms (as has been done), we might reach different conclusions.

Formalism is the principal alternative to Platonism. Under Plato, the truths and objects of mathematics *are there* somewhere, waiting to be discovered. The theorems are already true; by proving them we are just demonstrating what is actually the case. Formalism maintains they aren't

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[21] Roger Penrose, *The Emperor's New Mind*, p.127.

[22] Ibid, p.147.

[23] Changeux and Connes, *Conversations*, quoted in M. Livio, *Is God a Mathematician?*, pp.9-10.

anywhere: we just invent them as we go along by deducing them from axioms we have ourselves chosen. This is the view espoused for instance by Kline:

> Mathematics is a human creation. Although most Greeks did believe that mathematics existed independently of human beings as the planets and mountains seem to, and that all human beings do is discover more and more of the structure, the prevalent belief today is that mathematics is entirely a human product. The concepts, the axioms, and the theorems established are all created by human beings in man's attempt to understand his environment, to give play to his artistic instincts, and to engage in absorbing intellectual activity.[24]

What is at stake? If Plato is right about mathematics, there really *exists* a world of (mathematical, at least) abstracts, to which we have ready access with our minds. Our notion of existence is broader than those who reject belief in God on quasi-scientific grounds are prepared to grant.

CRITIQUE OF FORMALISM

In practice, formalism is seldom what actually happens.[25] Mathematicians do not generally sit down with a set of axioms in front of them in order to see what follows. Far more often, they work at the 'business end' of mathematics, and tie up the nuts and bolts (if at all) afterwards. So in the late seventeenth century, both Newton and Leibniz independently came up with the Calculus. It was not for another 160 years that the Calculus was made rigorous by *analysis* (by Cauchy, 1821-3, and others).

Again, in 1665-6, while *en route* to the Calculus, Newton discovered the binomial theorem (for all rational exponents), which in his hands and those of Euler and countless others proved to be a veritable powerhouse of mathematics. Yet, as with the Fundamental Theorem of the Calculus, he did not prove it, and indeed it did not receive its formal proof until Euler. It would be a curious doctrine of mathematics according to which Sir Isaac Newton was not a mathematician.

Indeed, the same cloud would hang over the Hindu mystic Srinivasa Ramanujan (1887-1920), perhaps the greatest intuitive mathematician of all time. According to Marcus du Sautoy,

> He used to claim that his ideas were given to him in his dreams by the goddess Namagiri, the Ramanujans' family goddess and consort of Lord Narasimha, the lion-faced, fourth incantation of Vishnu. Others in Ramanujan's village believed that the goddess had the power to exorcise demons. For Ramanujan himself she was the explanation for the flashes of insight that sparked his continuous stream of mathematical discoveries....
>
> Illumination itself was sufficient for Ramanujan. He just did not see the point in verification. Perhaps it was not having the responsibility of proof around his neck that allowed Ramanujan the freedom to discover new pathways through the mathematical wilderness. This intuitive style was quite at odds with the scientific traditions of the West. As Littlewood said later, 'The clear-cut idea of what is meant by a proof he did not possess at all; if the total mixture of evidence and intuition gave him certainty, he looked no further.' (*Music of the Primes*, p. 133-4)

Ramanujan was in love with what is. Hence in his case, as an account of what mathematicians

[24] M. Kline, *Mathematics for the Nonmathematician*, p.27.

[25] So R.G.D. Allen, *Basic Mathematics* (London: Macmillan, 1962), pp.2-7, gives an excellent, mainline description of the formalist 'axiomatic approach' to modern mathematics, only to undermine it fatally in a concluding footnote (p.7): 'A formal system developed strictly on the axiomatic method would be highly symbolised and a rather arid affair. Here we compromise in order to provide a general exposition which explains what is going on. It might be described as "informal axiomatics".' Any philosophy of mathematics which generates an introduction to the subject that is *boring* cannot possibly be right!

actually do, formalism with its emphasis on proof rather misses the point. As with Newton, the loss of Ramanujan would seem a very high price to pay for a dogma.

Next, formalism cannot explain the massively documented experience of mathematicians that in practice they do occupy a *shared world* in which their concepts are really there to be examined by them and their colleagues. They *assume* the reality of their subject matter whenever they communicate with their fellows, even if they cannot justify it. Otherwise they would have nothing in common to talk about.

Such concepts include mathematical *structures* such as finite simple groups. Mathematicians have spent years classifying these as lepidopterists classify butterflies.[26] Again, mathematicians investigating the Riemann hypothesis, by computing the zeros of the zeta function, have uncovered an infinite and unpredictable mathematical 'landscape' of hills and valleys, millions of which are being investigated every day by computer in search of a particular (ir)regularity.[27] It seems a little odd to maintain that such things in no way 'exist'. If they did not, why would anyone bother?

Formalists seem seldom to examine the philosophical status of the axioms upon which their beliefs depend. So the paradox of how a system claimed by its proponents to be devoid of meaning nevertheless provides us with so powerful a tool for understanding the universe remains without a satisfactory answer. What we have described as the mystery of mathematics has not been abolished; it has merely been relocated.

As an alternative to Platonism, formalism fails because it does not describe what mathematicians actually do. Further, it offers us no escape from the phenomenon of mystery.

THE CASE FOR PLATONISM

Can Platonism be made good in its own right? I shall argue that

(a) Numbers have properties.
(b) Theorems are universal in time and space.

(a) Numbers Have Properties

We have just argued that some mathematical structures lay a claim to 'existence'. The formalist would not agree, since these are defined by *axioms* which have been selected by *people*. They have no external reference. Suppose we grant this. Are there any areas of mathematical research which are *not* dependent upon axioms? Orthodoxy says no, but I think there are: *numbers*. The concept of number is I believe fundamental, as Poincaré held, in spite of attempts by Frege (1884) and others to define numbers in terms of sets. *Numbers have properties* in their own right which are to a large extent independent of axioms. For instance:

(1) *Prime numbers* - whole numbers (greater than 1) which have no factors other than 1 and

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[26] Philip J. Davis and Reuben Hersh, op. cit., pp.203-09.

[27] See Marcus du Sautoy, op. cit.

themselves - simply are what they are by their own very nature. They are probably the most intriguing and most intensely researched topic in mathematics today.[28] The classic statement of this again comes from Hardy:

> Pure mathematics...seems to me a rock on which all idealism founders: 317 is a prime, not because we think so, or because our minds are shaped in one way or another, but *because it is so*, because mathematical reality is built that way.' - (op. cit. p.130; emphasis original.)

Are prime numbers dependent upon a prior set of arithmetical axioms? I think, only trivially. Of course it is possible to reverse engineer axiom sets from which the notion of primality can be deduced. Several have been proposed. But historically the concept was arrived at by other means. Euclid had as clear a concept of primality as any of us long before anything like the present day set of arithmetical axioms was formulated.[29] And there is all the difference in the world between being able to define primality and being able to tell which numbers are prime and which composite. To understand primality you consult your axiom set. To know whether a given number is prime or not, you examine the number. With high numbers this can take vast amounts of computation.

Today one can join GIMPS - the Great Internet Mersenne Prime Search - which is a network of Personal Computers which together explore increasingly high Mersenne numbers in order to discover higher and higher primes.[30] Like much mathematical research today, this is an empirical inquiry into what is. It is not an explanation of the meaning of the concept 'prime'.

(2) *Constants* such as π and *e* are, as stated above, phenomenally important to all manner of mathematics. They crop up everywhere. Without them much of mathematics would be impossible. Since they are irrational, their decimal expansion never terminates. In fact, billions of digits of π have been computed. Further, whichever axiom set one adopts for any branch of mathematics, their values are unchanged. They are a mathematical given, forced upon us. They are what they are. Being therefore independent of axioms in any non-trivial sense, they constitute in themselves a powerful refutation of formalism. Likewise Kline's view that mathematics is a purely human invention falls to the ground when we ask, which human being determined that π and *e* should have the values that they do? (This is not the same as asking who first tried to calculate them, for which Archimedes and Newton respectively would be strong candidates.) If their values are not objectively determined by factors beyond the human ambit, as under Platonism, why are we unable to change them at will?

This takes us to bedrock. Numbers are at best only trivially dependent upon axioms, yet have immutable and objective properties in their own right which can be investigated.[31] This is illustrated by the well-known story of Hardy's visit to the ailing Ramanujan. Hardy was, recounts du Sautoy,

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[28] See David Wells, *Prime Numbers: The Most Mysterious Figures in Math.*

[29] Euclid's proof that there is an infinite number of primes stood unrivalled for some two thousand years as the most important theorem in number theory.

[30] A Mersenne number is a number of the form $2^n - 1$, where n is a positive integer. They are named afer the French mathematician and monk Marin Mersenne (1588-1648), who first explored them.

[31] See David Wells' fascinating compendium, *The Penguin Dictionary of Curious and Interesting Numbers*. Not many of them in my judgement owe their inclusion to prior axioms.

> unable to come up with any comforting sentiments. Instead, he offered the number of the taxi in which he arrived, 1,729, as an example of rather a dull number. Even on his sickbed, Ramanujan was unstoppable. 'No, Hardy! No, Hardy! It is a very interesting number. It is the smallest number expressible as the sum of two cubes in two different ways.' He was right: $1{,}729 = 1^3 + 12^3 = 10^3 + 9^3$. (*Music of the Primes*, p.145)

Hence, there must be a mode of existence in which it can be said that *numbers exist* in a significant, non-trivial way.

This conclusion is hotly contested by Professor Timothy Gowers in his booklet *Mathematics: A Very Short Introduction*. He argues (p.18) that 'A mathematical object *is* what it *does*', in the same way in which the black king in chess is best defined by the rules which specify how it moves. But his analogy fails spectacularly. If planet Earth were suddenly destroyed by an asteroid, there would be, as far as we know, no more black chess kings anywhere in the universe. But the properties of numbers would remain unaltered. 2 would still be the smallest, and the only even, prime number. The ratio of the circumference of a circle to its diameter would still be approximately 3.14159. The value of *e* likewise would not vary. The infinite series which define them would be unaffected.

(b) Theorems are Universal

It is inherent in mathematics that any theorem, once proved, is known to be universally valid. We do not have to travel to the moon or to the far side of the universe in order to retest its validity there: we *know* it will be valid. (We may pause to reflect *how* we know this; but know it we do.) *Mathematics is independent of place.*

Correspondingly, mathematical truths, if true at all, will have been true before the big bang (forgive the inadequacy of language) and will go on being true after the 'big crunch' (or whatever else ends the universe): *they are timeless,* with no beginning or end, *and actually independent of the universe altogether.* They would have been true had there never been a universe.

Any attempt to locate the concepts of mathematics in the minds that share them[32] runs instantly into the problem of continuity: what happens when the owners of those minds die? Do their theorems cease to be true?

Mathematics is therefore discovered, not invented. Plato was right. We may contrast *language*, which by common consent exists purely by convention. Developments in language have no known implications anywhere in the universe beyond planet Earth. The same could be said of the universal language that we call music.

MATHEMATICS A MODEL FOR GOD

So in the nature of numbers and in the universal quality of theorems we find support for the Platonic belief in an abstract world of mathematics which fInds regular expression among mathematicians. In a meaningful sense, the objects of mathematics *exist* - infinite in extent, eternally present in space and time and universally accessible to anyone prepared to use their mind. So just as mathematics provides us with a model for the mystery of God, so also does it offer us one for His eternal existence.

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[32] As Davis and Hersh, op. cit., pp.397-9.

Further, belief in mathematics is a necessary starting point for the advancement of science. This raises the possibility that science itself is an act of faith - for instance in the rationality of the universe and in its intelligibility to the human mind - in which belief in God may be not so much a contradiction or conflict as a stepping stone. This in turn offers us an interpretation of St Augustine's famous dictum '*Credo ut intellegam*' - 'I believe, that I may understand.'

There are two other qualities which mathematics shares with the God of the great monotheisms. First, both are widely held to be infinite in compass: you can never get to the end of either. Second, beauty, leading to awe and delight in the beholder. Just as the psalmist longs to behold the beauty of the Lord (27:4), so Hardy writes

> The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test [of good mathematics]: there is no permanent place in the world for ugly mathematics....
>
> It would be difficult now to find an educated man quite insensitive to the aesthetic appeal of mathematics. It may be very hard to *define* mathematical beauty, but that is just as true of beauty of any kind - we may not know quite what we mean by a beautiful poem, but that does not prevent us from recognizing one when we read it. (*A Mathematician's Apology*, p.85)

I recall a colleague who was converted to mathematics when he first discovered that most beautiful and surprising of all equations,

$$e^{i\pi} = -1$$

which follows directly from Euler's identities cited above. One could write an entire book about that equation, and all that it carries with it.

The delight with which mathematics inspires in its captivated worshippers has seldom been better exemplified than in the life of the perpetually boyish 'mathematical monk', Hungarian-born number theorist Paul Erdös (1913-1996), whose love of the subject, and the happiness it inspired in him, were simply infectious.[33] But I doubt if it has ever been better expressed in print than by H. E. Huntley's fascinating book *The Divine Proportion: A Study in Mathematical Beauty*, from which I quote just one illustration:

> A sense of wonder, even of awe, in the presence of the infinite, is one of the basic human emotions. Through all the aeons of time when man has stood beneath the cold light of stars and gazed into the unbounded depths of space; and especially since man first understood, a century ago, that an age-long stretch of evolutionary history lies behind him, infinity has been for him an emotionally charged concept. Music has power to arouse his emotion. So has mathematics. A divergent series of any sort induces this sense of infinity even as a convergent series leads to the related idea of the infinitesimal. Both feelings are roused by the spectacle of the curve of the hyperbola streaking off to infinite distance, simultaneously reducing its separation from its asymptote without ever reaching it. These are aspects of the aesthetic experiences of mathematics which easily pass unnoticed as such. (p.87)

But Huntley insists that appreciation of such beauty is only partly innate; beyond that, some

[33] See his biography *The Man Who Loved Only Numbers*, by Paul Hoffman.

personal application is required. As he puts it, 'Spade work is essential: *per ardua ad astra*.' (p.2)

Dawkins has almost nothing to say about mathematics and it would appear that he has done very little thinking about it. While this would be odd in a Professor for the Public Understanding of Science, it could explain the sheer naivety of his gung-ho comments about God.[34]

IS GOD A MATHEMATICIAN?

We have now arrived at a God whose possibility can be taken seriously by any intellect, and who is about as vulnerable to the assaults of Dawkins's peashooter as planet earth is to the passage of a neutrino. Dawkins's problem - apart from being a scientist in a philosopher's world - turns out to have been that his targeted God was simply too small. The possibility that God is mystery, combining as mathematics does the necessary and the contingent, and is at least as real as we have found the world of mathematics to be, now prompts us to enquire whether we have any positive reasons to believe in His existence. Is it pure coincidence that there is so much that is God-like in maths, or is there in fact a God who has quite a lot that is maths-like in Him? As Sir James Jeans wrote,

> From the intrinsic evidence of his creation, the Great Architect of the Universe now begins to appear as a pure mathematician.[35]

I recall the words attributed to Goldfinger by Ian Fleming in his James Bond novel of that name and quoted at the head of the Contents page:

> "Mr Bond, they have a saying in Chicago: 'Once is happenstance, twice is coincidence, the third time it's enemy action.'"

Or in the modern idiom, if it looks like a duck, waddles like a duck and quacks like a duck, it probably is a duck. I offer some thoughts.

First, this is a *personal* universe. It contains personality and consciousness. Each of us knows that there is at least one being that can say 'I am' correctly and without abusing language. Each of us can. Further, most of us believe there are others who can do the same. Now that is remarkable. On a purely reductionist view of science, this ought not to be possible. Where was personality at the big bang? What were its atomic or subatomic constituents? At what point did not just consciousness, but even self-consciousness arise, and what is it made of? This is not just a biological problem. You and I are more than biology. We are *people*. The simplest, neatest and least messy solution - the one that wins under Occam's Razor[36] - is that the universe is personal today because it has always been personal. We are here because there is Someone There. That would account in principle for all that we know.

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[34] See for instance the quotation from Douglas Adams, 'Isn't it enough to see that a garden is beautiful without having to believe that there are fairies at the bottom of it too?', to which Dawkins gives pride of place. John C. Lennox, Professor of Mathematics at Oxford University, comments, 'Fairies at the bottom of the garden may well be a delusion, but what about a gardener, to say nothing about an owner? The possibility of their existence cannot be so summarily dismissed - in fact, most gardens have both.' (*God's Undertaker*, p.40). On Dawkins generally, see Alister McGrath, *The Dawkins Delusion.*

[35] Sir James Jeans, *The Mysterious Universe* (1930), chapter 5.

[36] Occam's Razor: *Entia non sunt multiplicanda praeter necessitatem* ('Entities are not to be multiplied beyond what is necessary'). It is attributed to William of Occam (*c*.1285-1349), who used similar expressions, but it probably preceded him. The force of it is that we should always choose the simplest explanation which fits all the facts.

Second, this is a *moral* universe. There is good and evil. We all know this and have at least a concept of the difference between the two. And even if we disagree at times as to where lie the boundaries between them, almost all of us would accept that certain things are right and certain other things wrong in an absolute sense. Child abuse is evil. The Holocaust was evil. To deny this defines one as being morally flawed, even sub-human. Conversely, we have no real difficulty when not playing philosophical games in acknowledging some things to be good - not just because they have survival value, but good in themselves. For myself, I never fail to be moved by the heroism which causes one person to give up his or her life for others. So the same questions arise. Where were good and evil at the time of the big bang? Where have they come from? How is it that we have absolutes today if originally there were none? And again, the simplest, least messy, most comprehensive solution is that there are absolutes today because there has always been One who is Absolute. That in itself does not tell us everything. The fact of Absolute Goodness does not of itself explain the origin of evil. What it does do is put in place a framework within which problems like that can at least be explored and grappled with. Atheism by contrast can give no reason why anything matters at all.

Third, the *eureka* moment of sheer delight that is regularly reported by mathematicians on making a new discovery now has a simple explanation. They are delighted because they have been given a glimpse of the God of Delights.

So what we find as we examine one issue after another is that the single hypothesis of God makes sense of them all. Under it, answers become possible.

The question then arises, Has God Himself ever broken into history and pierced the veil that separates us from Him? Is it not reasonable to expect that the God we have been talking about in this paper might make some attempt to communicate Himself to us? The three great Abrahamic faiths maintain that the Hebrew Bible, known to Christians as the Old Testament, is just such a record of His attempting to make Himself known to fallible human beings. For me the most stunning proclamation in this book occurs when God discloses His Name to Moses as YHWH, 'the LORD' - 'I AM', and even 'I AM WHO I AM' (Exodus 3:14-15). How astonishingly appropriate for one who is both necessary and contingent! 'I AM WHO I AM' is both a necessary truth - true in virtue of its meaning, the very words used - and a contingent fact about the universe - He really does exist. This declaration seems to me to be every bit as stunning in its field as $e^{i\pi} = -1$ is in its.

Christians go further and maintain that the Old Testament is but the prelude to God's greatest revelation of Himself in the Babe of Bethlehem. Lord Hailsham - then Quintin Hogg - put it like this in his poem 'Song for the Nativity'. After questioning what the God of both galaxies and atoms can possibly make of the human race, he goes on to answer,

> 'Be still then and know; at the heart of the mystery
>   Hear in a manger an infant's cry.
> Hear and rejoice, at a moment of history
>   God made his answer: "Fear not, it is I"'.[37]

[37] Quintin Hogg, *The Devil's Own Song and other poems*, 39.

A well-known but anonymous text describes what follows, concluding,

> 'Nineteen centuries have come and gone, and today he is the central figure of the human race....
>
> 'All the armies that ever marched, all the navies that ever sailed, all the parliaments that ever sat, all the kings that ever reigned, put together, have not affected the life of man upon this earth as has that one solitary life.'[38]

To Him be glory for ever and ever. Amen.

Martin Mosse,
June 2011.

[38] Anon, 'One Solitary Life'.

ANNEX A

MYSTERY, PERSONALITY AND NUMBER

In this annex I offer some personal reflections on the nature of God as I perceive Him which some readers may find helpful, others not. If they help anyone, well and good. If not, nothing is lost.

In my own imagination three attributes of God stand out above all others. They are

Mystery: This includes the awesomeness of God, the fact that He lies so far beyond us and our comprehension, including His ability to hold together all things (Colossians 1:17) and to combine apparent opposites such as necessary / contingent and masculine / feminine without losing His integrity.

Personality: This includes His love, His moral qualities and His ability to communicate.

Number: This includes the entire body of mathematics through which He created the universe, and all logic and reasoning.

I am not suggesting that this threefold classification corresponds to the three Persons of the Trinity; rather, that all three Persons all embody all three attributes.

I am guided in my thinking by Anne Moir and David Jessell's book *Brainsex: The Real Difference Between Men and Women* which argues that the essential differences between the sexes arise from hormone levels in the womb during pregnancy, resulting in characteristic differences in the way the brain is wired. Not least, it is typical for girls to acquire language and communication skills more easily than boys (e.g. p.58), while boys, who characteristically have stronger visuo-spatial skills, tend to excel more in mathematics and reasoning, especially after puberty (e.g. p.89). (I am aware that these findings are highly controversial, but am encouraged to take them seriously by the sixteen and a half densely typed pages of references at the end giving the scientific support as it then stood. The thesis at least looks plausible.)

There seems to be a parallelism. So perhaps 'Personality' within the godhead encapsulates the feminine attribute of communication, while 'Number' characterises the masculine strength in logic and reasoning. Of course there will be exceptions and overlaps in all this, but I find the patterns involved to be helpful.

I now turn to my 1881 abbreviated Liddell and Scott's Greek-English Lexicon under the entry *logos*, which has two essential clusters of meaning. The first corresponds to the Latin *oratio* or *vox*, meaning that which is said or spoken; a word, saying, expression, conversation or discussion. It is all about communication. The second corresponds to Latin *ratio*, thought, reason, calculation or reckoning. It is all about reasoning. Special mention is then made of the New Testament - with the early verses of St John's Gospel clearly in mind[39] - where *LOGOS* is expressly stated to comprise both senses of *Word* and *Reason*. This suggests to me that St John's *Logos* embodies both the feminine

[39] 'In the beginning was the *Logos*, and the *Logos* was with God, and the *Logos* was God.' (John 1:1)

characteristics of God that I have dubbed ‘Personality’ as well as His masculine ones (‘Number’); while because God is Mystery there is no conflict between the two. Rather, the two facets are seen combined together in the person of Christ.

One benefit of this classification is that it offers us an explanation for the way mathematics supplies us with a model for God as argued in our main text. The suggestion that God is ‘Number’ includes the possibility that *mathematics is one of the ways in which God thinks*. As Ramanajan put it, ‘An equation means nothing to me unless it expresses a thought of God.’[40] So when the Bible says that

> ‘By the word of the LORD the heavens were made,
> and all their host by the breath of his mouth’ (Psalm 33: 6 RSV)

and the contemporary scientist says that the universe was generated in accordance with the laws of mathematics, they are simply using different languages appropriate to their separate domains of interest.

Continuing this line of approach, many people respond more readily to one aspect of God than to others. So the Hebrews’ God was very personal: ‘I will walk among you, and will be your God, and you shall be my people’ (Leviticus 26:12 RSV). But the Hebrews of those days were no great mathematicians. The implied value of 3 for π given at 1 Kings 7:23 is the least accurate that has come down to us from any ancient civilisation. By contrast, the Greeks enjoyed no such covenant relationship with God as the Hebrews did - think of the altar ‘To God Unknown’ that Paul found at Athens (Acts 17:23) - but produced an astonishing number of great mathematicians and indeed were the first people on record to have turned mathematics into the regular discipline we know today. But Paul sees Jew and Greek as complementary types of humanity and proclaims Christ as the Saviour and fulfilment of both (Galatians 3:28). And on the twin foundations of the essentially religious Judaeo-Christian, and the essentially humanistic, classical Graeco-Roman traditions has been built the whole of Western civilisation.

Further, the great Oriental faiths and the Eastern (Orthodox) Churches seem to me to be particularly strong on God’s Mystery, by contrast with the Reformation Churches and their present day evangelical manifestation, who are strong on the Personal (‘What a Friend we have in Jesus’) but weak on Mystery. I attribute this last point to the rejection by the Reformers of the mystical tradition, and of the contemplative silence and solitude by which one approaches Mystery. I have written of this elsewhere.[41]

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[40] du Sautoy, op. cit. p.132.

[41] BW/010 ‘Healing of the Nation’ and BW/012 ‘Healing of the Church.’

ANNEX B

WHY WE NEED METAPHYSICS

The Islamic (Sufi) writer Seyyed Hossein Nasr, in his penetrating, prophetic book *Man and Nature: The Spiritual Crisis of Modern Man*, presents a powerful argument on the need for Western civilisation, and in particular the Western Churches, to return to its own traditional metaphysical base. Upon the departure from this base he blames the present worldwide environmental crisis. And he traces the beginning of this departure to the Renaissance. I offer here some key quotations which largely speak for themselves.

> One of the chief causes for this lack of acceptance of the spiritual dimension of the ecological crisis is the survival of a scientism which continues to present modern science not as a particular way of knowing nature, but as a complete and totalitarian philosophy which reduces reality to the physical domain and does not wish under any condition to accept the possibility of the existence of non-scientific world views. (p.4)
>
> Modern man, faced with the unprecedented crisis of his own making which now threatens the life of the whole planet, still refuses to see where the real causes of the problem lie. He turns his gaze to the Book of Genesis and the rest of the Bible as the source of the crisis rather than looking upon the gradual de-sacralization of the cosmos which took place in the West and especially the rationalism and humanism of the Renaissance which made possible the Scientific Revolution and the creation of a science whose function, according to Francis Bacon, one of its leading proponents, was to gain power over nature, dominate her and force her to reveal her secrets not for the glory of God but for the sake of gaining worldly power and wealth. (p.6)
>
> Having devastated nature through the application of a science of a purely material order combined with greed, modern man now wishes to put the blame at the door of the whole Western religious tradition. But because the reality of the Spirit is such that it cannot be denied by any form of sophism or limited science of the material order, the ecological crisis cannot be solved without paying particular attention to the spiritual dimension of the problem. (p.7)
>
> ...the ecological crisis is only an externalization of an inner malaise and cannot be solved without the spiritual rebirth of Western man. (p.9)
>
> There is everywhere the desire to conquer nature, but in the process the value of the conqueror himself, who is man, is destroyed and his very existence threatened. (p.19)
>
> The disappearance of a real cosmology in the West is due in general to the neglect of metaphysics, and more particularly to a failure to remember the hierarchies of being and of knowledge. (p.23)

He blames Protestant theologians in particular for concentrating

> on the question of the redemption of man as an isolated individual rather than on the redemption of all things. (p.31)

And referring to Romans 8, he adds,

> The total salvation of man is possible when not only man himself but all creatures are redeemed. (p.34)

Also,

> The knowledge of the whole Universe does not lie within the competence of science but of metaphysics. (p.35)

Of particular interest to me is his detection of a ‘movement from the contemplative to the passionate' (p.37).

> The Middle Ages thus drew to a close in a climate in which the symbolic and contemplative view of nature had been for the most part replaced by a rationalistic view. (pp.63-4)

This chimes in very well with two earlier papers in this series, BW/010 'Healing of the Nation' and BW/012 'Healing of the Church', in which I lamented the fact that the Protestant Churches were born without a contemplative dimension.

I draw three conclusions. For the good of the planet:

(1) The West urgently needs to rediscover its metaphysical roots. In terms of the hierarchies of knowledge to which Nasr refers, I know of no better place to start than E. F. Schumacher's excellent book, *A Guide for the Perplexed*.[42]

(2) The Western Churches, and Protestants in particular, equally urgently need to rediscover in contemplative prayer the mystical tradition.

(3) Atheism is a luxury that this planet cannot afford.

[42] There is a review of this in Section IV of this website (see Bibliography).

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